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Tangential Winding Coil Probes for Dipole, Quadrupole and Sextupole Magnet Measurements

1. Introduction

The number of coils for a rotating coil probe of "tangential winding geometry" depends on the requirements of the magnet measurements. In order to measure the main field, multipole coefficients and the location of the magnetic center axis, two sets of coils are sufficient for a rotating coil probe of "radial winding geometry." A probe of tangential winding geometry, on the other hand, requires more than two sets of coils to measure the above field parameters. This note describes probes of tangential winding geometry with a minimum number of coils for dipole, quadrupole and sextupole magnet measurements.

2. Two-dimensional Magnetic Field

The 2-D field in the aperture of a magnet may be expressed as

$$B_y + iB_x = B_o \sum_{m=o}^{\infty} (b_m + ia_m)(x + iy)^m, \tag{1}$$

or

$$B_\theta + iB_r = \sum_{n=1}^{\infty} C_n \exp(-in\alpha_n) \left(\frac{z}{R}\right)^{n-1} \exp(i\theta), \tag{2}$$

where $z = x + iy = r\exp(i\theta)$ and R is a reference radius. The multipole coefficients C_n, b_n and a_n are related to each other as

$$\frac{C_n}{R^{n-1}} \exp(-in\alpha_n) = B_o\,(b_{n-1} + ia_{n-1}). \tag{3}$$

In Eq. (1) it is defined that $b_o = 1.0$ for dipole, $b_1 = 1.0\ \mathrm{cm}^{-1}$ for quadrupole and $b_2 = 1.0\ \mathrm{cm}^{-2}$ for sextupole magnets.

3. Dipole Magnet Probe

Figure 1 shows the cross section of a probe configuration for dipole measurements. It consists of two coil sets: one Δ-coil with an opening angle of Δ and one π-coil (a dipole coil with an opening angle π).

The flux linkage of the two coils at an angular position of θ is given by

$$\phi_\Delta(\theta) - \phi_D(\theta) = LN_\Delta \sum_{n=1}^{\infty} C_n \left(\frac{r}{R}\right)^{n-1} \frac{r}{n} 2\sin n(\theta - \alpha_n) * \left[\sin\left(\frac{n\Delta}{2}\right) - (N_D/N_\Delta)\sin\left(\frac{n\pi}{2}\right)\right], \tag{4}$$

where ϕ_Δ and ϕ_D are flux linkages of the Δ-coil and π-coil respectively, L the effective magnetic length of the measuring magnet, and N_Δ and N_D are the number of turns of the Δ-coil and π-coil.

The condition for the bucking of the dipole field component is

$$N_\Delta \sin\left(\frac{\Delta}{2}\right) - N_D = 0. \tag{5}$$

With a choice of the probe coil parameters, $N_\Delta = 14$, $N_D = 3$ and $\Delta = 24.74725°$, relative sensitivities of multipole coefficients are listed in Table 1.

4. Quadrupole Magnet Probe

Figure 2 shows the cross section of a probe configuration for quadrupole magnet measurements. The probe consists of one Δ-coil, one π-coil and two $\pi/2$-coils (quadrupole coils).

The flux linkage of the probe at an angle θ is given by

$$\phi_\Delta(\theta) - \phi_D(\theta) - \phi_Q(\theta) = LN_\Delta \sum_{n=1}^{\infty} C_n \left(\frac{r}{R}\right)^{n-1} \frac{r}{n} 2\sin n(\theta - \alpha_n) * \left[\sin\left(\frac{n\Delta}{2}\right) - (N_D/N_\Delta)\sin\left(\frac{n\pi}{2}\right) - \{1 + (-1)^n\}(N_Q/N_\Delta)\sin\left(\frac{n\pi}{4}\right)\right], \tag{6}$$

where ϕ_Q is the flux linkage of the two quadrupole coils each with N_Q turns.

The bucking conditions of the dipole and quadrupole fields components of the probe coil in Eq. (6) are

$$C_1 \left[N_\Delta \sin\left(\frac{\Delta}{2}\right) - N_D\right] = 0, \tag{7}$$

$$C_2 \left[N_\Delta \sin(\Delta) - 2N_Q\right] = 0. \tag{8}$$

With a choice of the probe coil parameters, $N_\Delta = 18$, $N_D = 3$, $N_Q = 3$, and $\Delta = 19.47122°$, relative sensitivities of multipole coefficients are listed in Table 2.

The implication of Eq. (7) is further discussed for the case when the rotating axis of the probe is displaced from the magnetic axis of the measuring magnet. When the rotating coil and magnet axes do not coincide as shown in Figure 3, the multipole coefficients with respect to the xy-coordinate system and those with respect to the x'y'-coordinate system are related as

$$\sum_{J=1}^{\infty} C_J' \exp(-iJ\alpha'_J) \left(\frac{Z'}{R}\right)^{J-1}$$

$$= \sum_{J=1}^{\infty} \sum_{N=J}^{\infty} C_N \exp(-iN\alpha_N) \frac{(N-1)!}{(N-J)!(J-1)!} \left(\frac{Z'}{R}\right)^{J-1} \left(\frac{Z_o}{R}\right)^{N-J}. \tag{9}$$

From Eq. (9) the field in the x'y'-coordinates for a quadrupole magnet C_n $(n \neq 2) << C_2$ is given by

$$B'_\theta + iB'_r = C_2 \exp(-i2\alpha_2) \frac{Z_o + Z'}{R} \exp(i\theta'). \tag{10}$$

The flux linkages of the coils in Fig. 2 due to Eq. (10) are

$$\phi_\Delta(\theta') = LN_\Delta \frac{C_2}{R} \left[r_o r\, 2\sin(\theta' + \theta_o - 2\alpha_2) \sin\left(\frac{\Delta}{2}\right) + r^2 \sin 2(\theta' - \alpha_2) \sin(\Delta)\right], \tag{11}$$

$$\phi_D(\theta') = LN_D \frac{C_2}{R} r_o r\, 2 \sin(\theta' + \theta_o - 2\alpha_2), \tag{12}$$

$$\phi_Q(\theta') = LN_Q \frac{C_2}{R} 2r^2 \sin 2(\theta' - \alpha_2). \tag{13}$$

Flux linkage, $\phi_\Delta(\theta') - \phi_D(\theta') - \phi_Q(\theta')$ of Eqs. (11), (12) and (13) satisfies the bucking conditions of Eqs. (7) and (8). In this case the dipole coefficient in Eq. (7) is equivalent to $C_1 = C_2\, r_o/R$.

As shown in Eq. (13) the flux linkage of the two $\pi/2$-coils does not depend on $r_o \exp(i\theta_o)$; it depends only on the quadrupole field strength of the magnet.

In summary, the quadrupole magnet probe has the following features.

1. In the measurements of multipole coefficients, the dipole and quadrupole components are cancelled out, whether the probe axis is located in the magnetic axis or not.

2. The detection of the magnetic center axis is expressed in Eq. (12). When the radii of the winding conductors of the π-coil differ by Δr, an additional term due to the quadrupole field,

$$LN_D \frac{C_2}{R} \Delta r \cdot r \sin 2(\theta' - \alpha_2)$$

   should be added to Eq. (12).

3. The quadrupole field integral is measured from the two $\pi/2$-coils as shown in Eq. (13).

5. Sextupole Magnet Probe

Figure 4 shows the cross section of a probe configuration for sextupole magnet measurements. The probe consists of one Δ-coil, two $\pi/2$-coils and two $\pi/3$-coils.

The coils are connected to have the flux linkage of the coils such that

$$\phi_\Delta(\theta) - \phi_Q(\theta) - \phi_S(\theta) = LN_\Delta \sum_{n=1}^{\infty} C_n \left(\frac{r}{R}\right)^{n-1} \frac{r}{n}\, 2 \sin n(\theta - \alpha_n) *$$

$$\left[\sin\left(\frac{n\Delta}{2}\right) - \{1 + (-1)^n\}\, N_Q/N_\Delta \sin\left(\frac{n\pi}{4}\right) - \{1 - (-1)^n\}\, N_S/N_\Delta \sin\left(\frac{n\pi}{6}\right)\right], \tag{14}$$

where $\phi_S(\theta)$ is the $\pi/3$-coil flux linkage and N_S is the number of turns for each of the $\pi/3$-coils.

The bucking conditions for the quadrupole and sextupole field components in Eq. (14) are

$$C_2\left[N_\Delta \sin(\Delta) - 2N_Q\right] = 0, \tag{15}$$

$$C_3\left[N_\Delta \sin\left(\frac{3\Delta}{2}\right) - 2N_S\right] = 0. \tag{16}$$

With a choice of the probe parameters $N_\Delta = 15$, $N_Q = 2$, $N_S = 3$ and $\Delta = 15.71879°$, relative sensitivities of the multipole coefficients are listed in Table 3.

When the probe axis is displaced from the magnetic axis as shown in Fig. 3, for a sextupole magnet $C_n(n \neq 3) << C_3$, one obtains from Eq. (9)

$$B'_\theta + iB'_r = C_3 \exp(-i3\alpha_3)\left[\frac{Z_o + Z'}{R}\right]^2 \exp(i\theta'). \tag{17}$$

The flux linkages of the coils in Fig. 4 due to Eq. (17) are

$$\phi_\Delta(\theta') = LN_\Delta \frac{C_3}{R^2}\Big[2r_o r^2 \sin(2\theta' + \theta_o - 3\alpha_3)\sin(\Delta) + \frac{2}{3} r^3 \sin 3(\theta' - \alpha_3) \sin\left(\frac{3\Delta}{2}\right)\Big], \tag{18}$$

$$\phi_Q(\theta') = LN_Q \frac{C_3}{R^2} 4r_o r^2 \sin(2\theta' + \theta_o - 3\alpha_3), \tag{19}$$

$$\phi_S(\theta') = LN_S \frac{C_3}{R^2} \frac{4}{3} r^3 \sin 3(\theta' - \alpha_3). \tag{20}$$

The flux linkage, $\phi_\Delta(\theta') - \phi_Q(\theta') - \phi_S(\theta')$ of Eqs. (18), (19), and (20) also satisfies the bucking conditions of Eqs. (15) and (16). The quadrupole coefficient in Eq. (15) is equivalent to $2r_o C_3/R$. The dipole field term in Eq. (17) which is proportional to $r_o^2 r$, is neglected in the subsequent calculations in Eqs. (18) ~ (20).

From Eqs. (19) and (20) the magnetic axis and sextupole field integral are measured.

Table 1. Probe Coil Parameters for Dipole Magnets.

$N_\Delta = 14$, $N_D = 3$, and $\Delta = 24.74725°$.

	A	B	C
1	Tangential coil probe for dipole magnet measurements		
2			
3	ndelta	14	
4	ndipole	3	
5	delta	24.74725	
6	pi	3.14159265	
7			
8			
9	n	Sensitivity	
10	1	-2.22181E-09	
11	2	0.418616148	
12	3	0.817784251	
13	4	0.76034362	
14	5	0.66757898	
15	6	0.962416139	
16	7	1.212541421	
17	8	0.987718359	
18	9	0.71700792	
19	10	0.831602913	
20	11	0.907563346	
21	12	0.522744093	
22	13	0.113639211	
23	14	0.117871036	

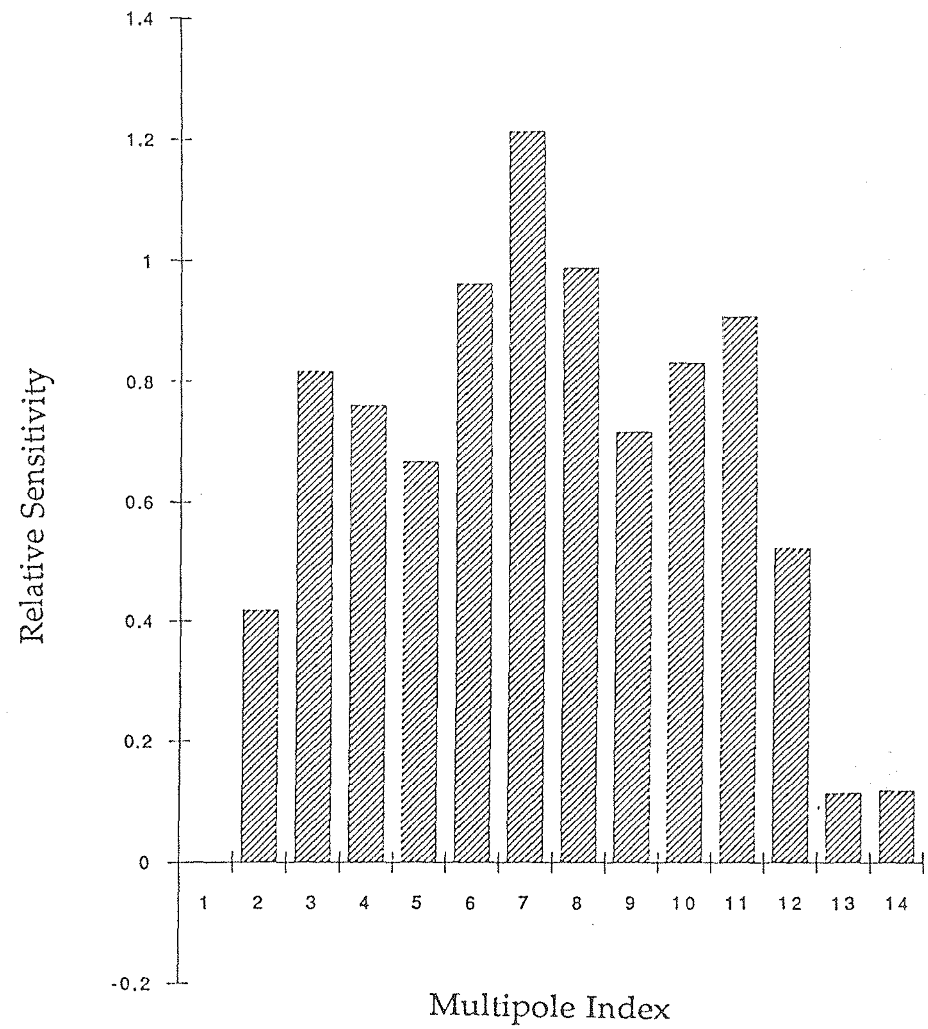


Table 2. Probe Coil Parameters for Quadrupole Magnets.

$N_{\Delta} = 18, N_{D} = 3, N_{Q} = 3$, and $\Delta = 19.47122^{\circ}$.

	A	B	C
1	Tangential coil probe for quadrupole magnet measurements		
2			
3	ndelta	18	
4	ndipole	3	
5	nquad	3	
6	delta	19.47122	
7	pi	3.14159265	
8			
9			
10	n	sensitivity	
11	1	0.002435306	
12	2	-1.1405E-08	
13	3	0.654630379	
14	4	0.628539343	
15	5	0.584344565	
16	6	1.185185165	
17	7	1.094823318	
18	8	0.97772789	
19	9	0.832471075	
20	10	0.658436218	
21	11	1.122502216	
22	12	0.892370729	
23	13	0.636536396	
24	14	1.024234164	
25	15	0.725365342	

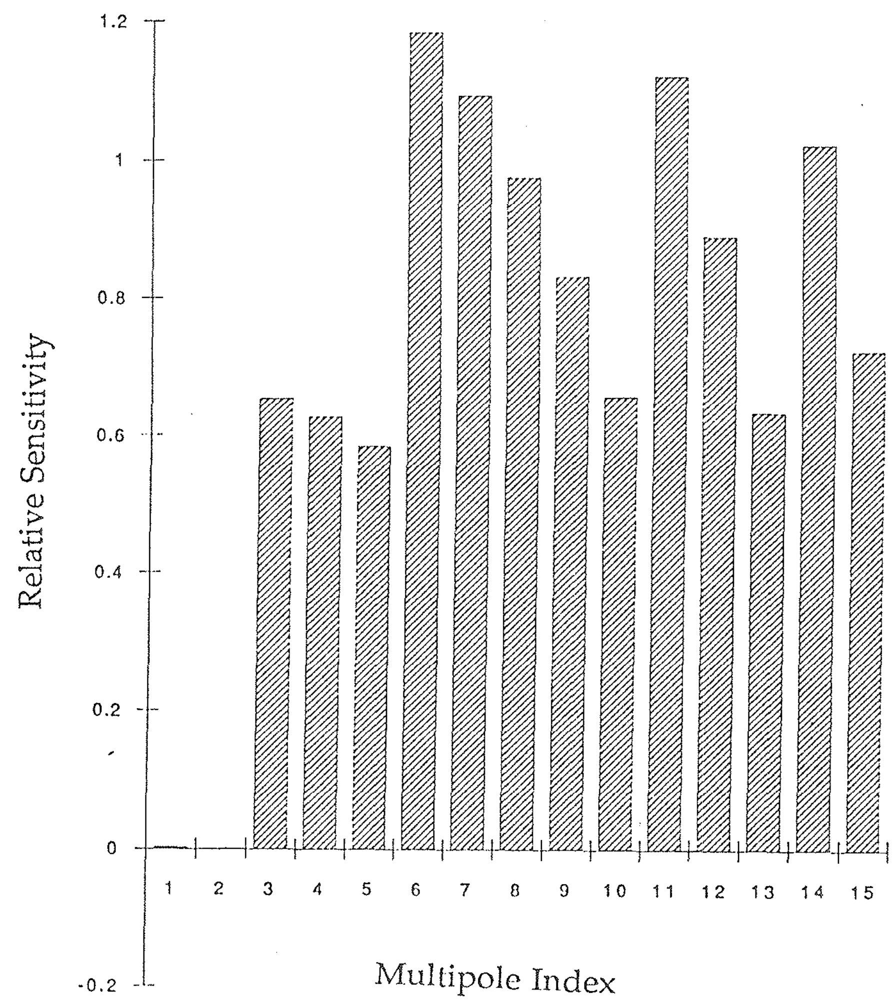
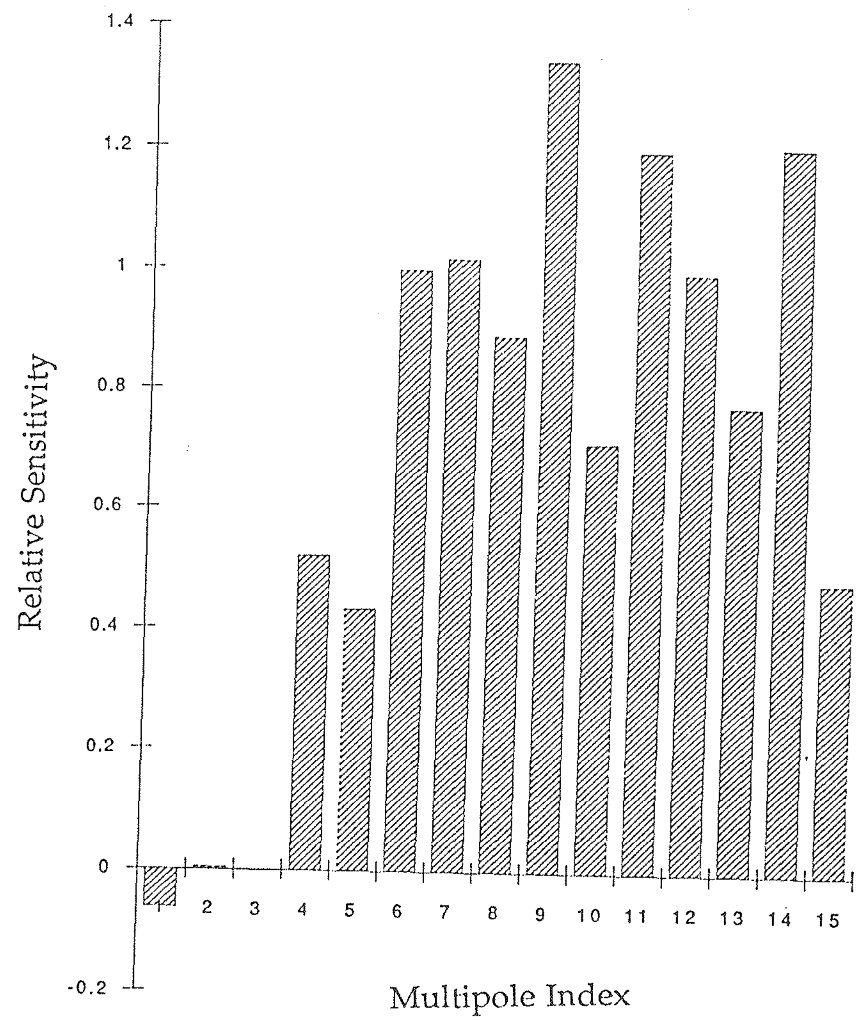


Table 3. Probe Coil Parameters for Sextupole Magnets.

$N_\Delta = 15$, $N_Q = 2$, $N_S = 3$, and $\Delta = 15.7187°$.

	A	B	C
1	Tangential coil probe for sextupole magnet measurements		
2			
3	ndelta	15	
4	nquad	2	
5	nsext	3	
6	delta	15.71879	
7	pi	3.14159265	
8			
9			
10	n	sensitivity	
11	1	-0.063257453	
12	2	0.004249477	
13	3	1.03893E-07	
14	4	0.521569358	
15	5	0.433340014	
16	6	0.999878932	
17	7	1.019309831	
18	8	0.890015225	
19	9	1.344000112	
20	10	0.713583631	
21	11	1.198084759	
22	12	0.997168434	
23	13	0.777518557	
24	14	1.206170931	
25	15	0.483839735	



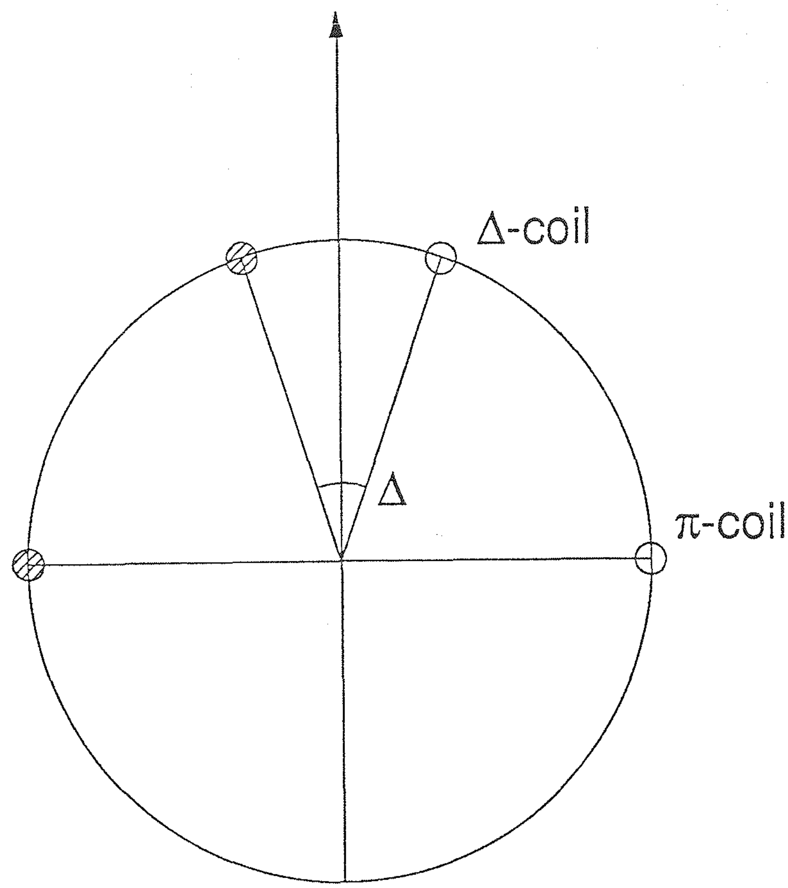


Fig. 1. Cross section of a tangential wing coil for dipole magnet measurements. It consists of one Δ-coil and one π-coil (dipole coil) which is used for the bucking of the dipole field and for the measurements of dipole field integral. The vertical axis, which bisects the angle of the Δ-coil is the reference angular direction, $\theta=0$.

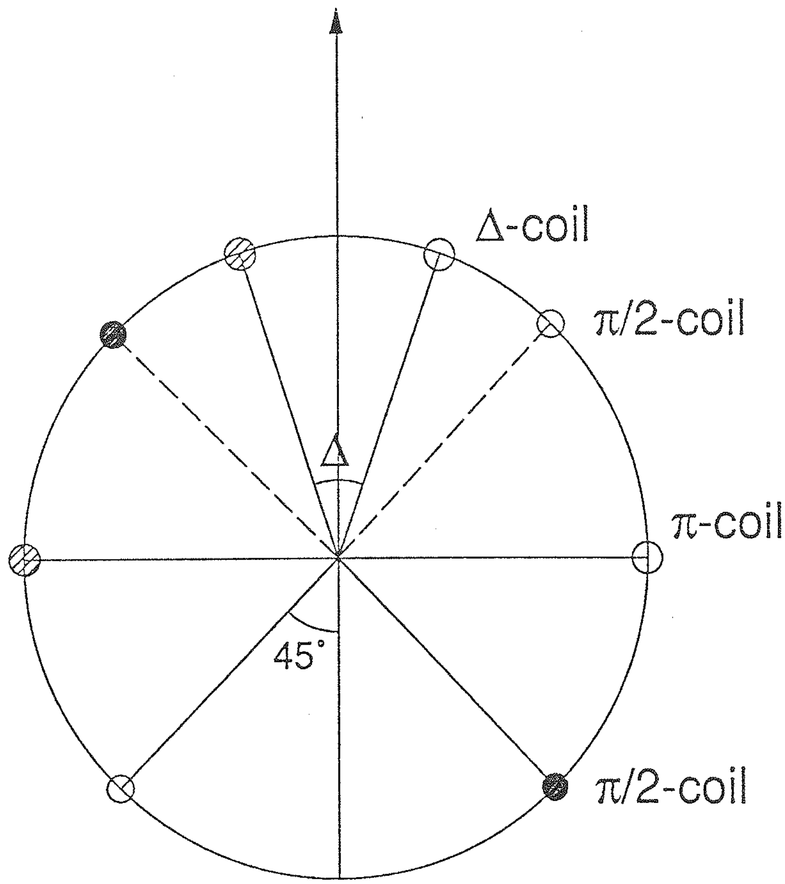


Fig. 2. Cross section of a tangential winding coil for quadrupole magnet measurements. It consists of one Δ-coil, one π-coil which is used for the bucking of the dipole field and for the measurements of the magnetic center, and two π/2-coils for the bucking of the quadrupole field and for the measurements of quadrupole field integral.

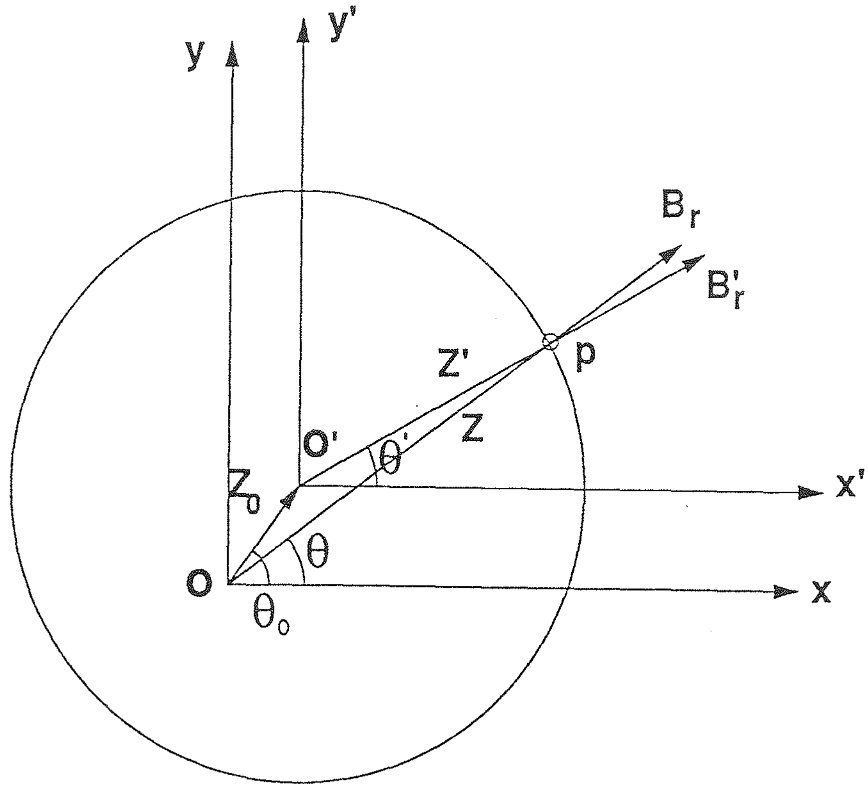


Fig.3. Coordinate systems of the magnetic center (MC) and cylinder rotation axis (CR). The MC is located at O. The CR,which is located at O',is displaced from MC by $Z_o = r_o \exp(i\theta_o)$. The corresponding axes of the xy and x'y' coordinates are in parallel. Z and Z' are coordinates of point P on the cylinder surface with respect to the two coordinate systems.

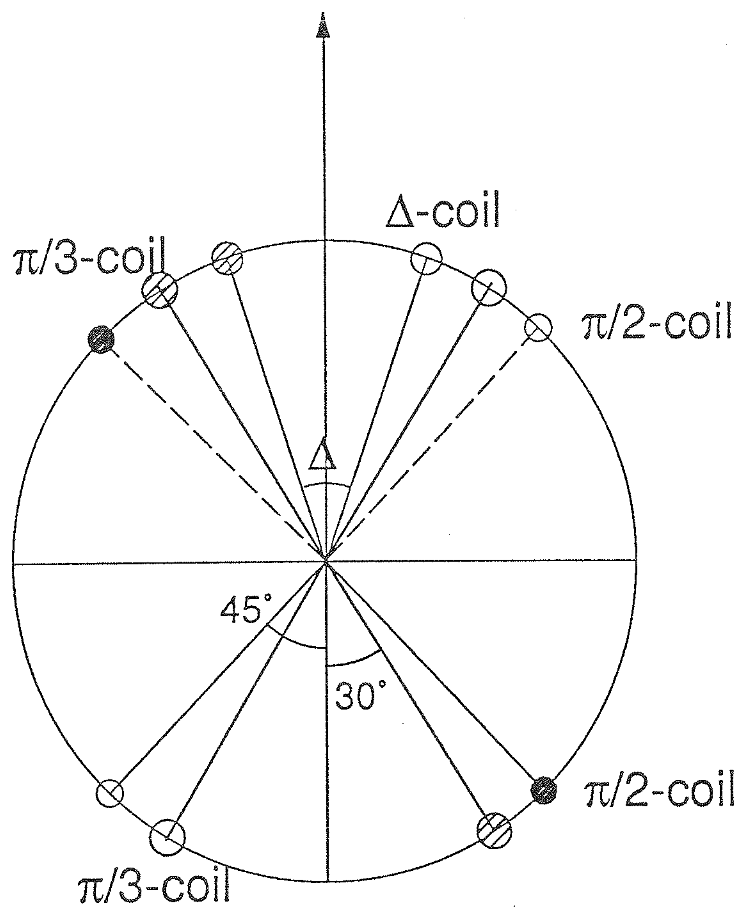


Fig.4. Cross section of a tangential winding coil for sextupole magnet measurements. It consists of one Δ-coil, two $\pi/2$-coils which is used for the bucking of quadrupole field and for the measurements of the magnetic center, and two $\pi/3$-coils which is used for the bucking of the sextupole field and for the measurements of the sextupole field integral.